1.4

Equation Solving and Modeling

<u>One to one properties</u>:

For any exponential function $f(x) = b^x$,

- If $b^u = b^v$, then $u = v$.

For any logarithmic function $f(x) = \log_b x$,

- If $\log_b u = \log_b v$, then $u = v$.

<u>Solving Exponential Equations:</u>

Solve: $20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$

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$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \left(\frac{1}{2}\right)^2$$

$$\frac{x}{3} = 2 \qquad x = 6$$

See example 2 pg. 321.

<u>Solving Logarithmic Equations:</u>

Solve: $\log x^2 = 2$.

$\log x^2 = 2 \quad \rightarrow \quad 10^2 = x^2 \quad \rightarrow \quad x = 10$ or -10

See methods 1 & 3 also pg. 321- 322.

Solve: $\ln(3x - 2) + \ln(x - 1) = 2\ln x$.

$\ln(3x - 2) + \ln(x - 1) = 2\ln x$

$\ln[(3x - 2)(x - 1)] = \ln x^2$

$(3x - 2)(x - 1) = x^2$

$3x^2 - 3x - 2x + 2 = x^2$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = ½$ or $x = 2$ check domain $x \neq ½$ so $x = 2$ is the only solution.

We can also solve this by graphing, setting the original equation equal to 0 and finding the x-intercepts.

Solve: $\ln(3x - 2) + \ln(x - 1) = 2\ln x$.

$\ln(3x - 2) + \ln(x - 1) - 2\ln x = 0$

x-intercept is at (2, 0).

Newton's Law of Cooling :

$T(t) = T_m + (T_o - T_m)\, e^{-kt}$

T_m = the temperature of the surrounding medium

T_o = initial temperature of the object.

See example 7 pg. 326.